

All equation, table, and figure numbers refer to the book ‘Fundamentals of Molecular Spectroscopy’ by Philip R. Bunker and Per Jensen.

- 1.1 What is the wavenumber, frequency and energy per photon of (a) visible radiation having a wavelength of 500 nm, (b) infrared radiation having a wavelength of 5  $\mu\text{m}$ , and (c) microwave radiation having a wavelength of 5 cm? Quote the frequency in appropriate units such as THz or GHz, and the wavenumber in  $\text{cm}^{-1}$ .
- 1.2 The successive rotational energy levels of the  $^{12}\text{C}^{16}\text{O}$  molecule depicted in figure 1.5 are labeled  $J = 0, 1, 2, \dots$ , and to three significant figures their energy divided by  $hc$  is  $E_i = 1.92J(J+1) \text{ cm}^{-1}$ ; the degeneracy of each level is  $g_i = (2J+1)$ . Calculate the numerator in the Maxwell-Boltzmann distribution function equation (1.5) for a range of values of  $J$  for temperatures  $T$  of 10, 300 and 1000 K to confirm the  $J$ -value at which the population is a maximum according to figure 1.7; in the appropriate units  $k \approx 0.695 \text{ cm}^{-1}\text{K}^{-1}$ . By setting equal to zero the differential of the numerator with respect to  $J$ , determine an expression as a function of  $T$  for the value of  $J$  at which the population is a maximum. Check that this leads to the correct result for  $T = 10, 300$  and 1000 K ( $J$  has to be an integer).

- 1.3 Translational motion of molecules gives rise to Doppler broadening of spectral lines. A Doppler-broadened line with central frequency  $\nu_0$  has a full width at half height FWHH (the frequency width of the line at half the maximum intensity) of

$$\begin{aligned} \text{FWHH} &= \frac{2\nu_0}{c} \left( \frac{2kT}{M} \ln 2 \right)^{1/2} \\ &\approx 7.15 \times 10^{-7} \left( \frac{T}{M/\text{u}} \right)^{1/2} \nu_0, \end{aligned} \quad (1)$$

where  $k$  is Boltzmann's constant,  $T$  is the absolute temperature,  $M$  is the mass of the molecule in question, and  $\text{u}$  is the unified atomic mass unit.<sup>1</sup> Calculate such FWHH Doppler widths of infrared absorption lines of the  $\text{H}_3^+$  molecule ( $M/\text{u} \approx 3$ ) and  $\text{CH}_4$  molecule ( $M/\text{u} \approx 16$ ) at around  $3000 \text{ cm}^{-1}$  at 1000 K and at 10 K. What would the Doppler widths be for lines of these molecules at these temperatures at wavelengths around 1 cm (in the microwave region) or 100 nm (in the ultraviolet region)? The appropriate units for linewidth are  $\text{cm}^{-1}$  in the infrared and ultraviolet, and kHz in the microwave.

- 1.4 We describe the rotation of the molecule  $^{79}\text{Br}^{19}\text{F}$  under the assumption that the molecule is rigid. The allowed energies of the corresponding rigid rotor are

$$E_J = h c B J(J+1); \quad J = 0, 1, 2, 3, \dots,$$

with the rotational constant

$$B = \frac{h}{8\pi^2 c \mu r_e^2}.$$

The quantity  $\mu$  is the reduced mass of the molecule and  $r_e$  is the constant internuclear distance.

Irradiation with light in the microwave region induces absorption transitions. An allowed transition from the state with the energy  $E_J$  ends in the state with the energy  $E_{J+1}$ . A progression of transitions are observed. The wavenumber interval between two neighbouring lines is independent of  $J$ . This interval is measured to have the value  $0.71433 \text{ cm}^{-1}$ . Calculate  $B$ , the moment of inertia

$$I = \mu r_e^2$$

and  $r_e$ . Calculate the wavenumber of the transition  $J = 9 \rightarrow J = 10$ .

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<sup>1</sup>1 u =  $1.660\,538\,86 \times 10^{-27}$  kg; also called the dalton or the atomic mass constant.